From the Bonnet problem to the Lax pair of $\mathrm{P_{VI}}$

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We build analytic surfaces in $\mathbb{R}^3(c)$ represented by the most general sixth Painlevé equation $\mathrm{P_{VI}}$, in two steps. Firstly, the moving frame of the surfaces built by Bonnet in 1867 is extrapolated to a new, second order, isomonodromic matrix Lax pair of $\mathrm{P_{VI}}$, whose elements depend rationally on the dependent variable and quadratically on the monodromy exponents θ_j. Secondly, by converting back this Lax pair to a moving frame, we obtain a generalization of Bonnet surfaces to two more degrees of freedom.

Reference:

R. Conte, Surfaces de Bonnet et équations de Painlevé,
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http://arxiv.org/abs/1607.01222v2